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Lower Bounds for Resonance Widths in Potential and Obstacle Scattering

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Abstract. Explicit lower bounds are given for the size of the imaginary parts of resonances for Schrödinger operators with non-trapping or trapping potentials, and for the Dirichlet Laplacian in the exterior of a star-shaped obstacle, both acting in three dimensions.

1. Introduction

Resonances for perturbations of the Laplace operator Δ on $\mathbb{R}^n$ are of interest in the theory of scattering for the Schrödinger equation

$$\frac{\partial\psi(x,t)}{\partial t} = -i(-\Delta + V(x))\psi(x,t) \quad x\in\mathbb{R}^n, t\in\mathbb{R} \tag{1.1}$$

and the wave equation outside an obstacle Ω

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \Delta u(x,t) \quad x\in\mathbb{R}^n\backslash\Omega, t\in\mathbb{R}. \tag{1.2}$$

They are associated with abnormally long, but temporary trapping of quantum mechanical particles for (1.1), or waves for (1.2). Mathematically, a self adjoint perturbation H of $-\Delta$ is said to have a resonance $k=\kappa - i\eta\in\mathbb{C}$ if its resolvent $(H-z)^{-1}$ has an analytic continuation in z with a pole at k^2. This gives a solution ψ of the eigenvalue equation $H\psi = k^2\psi$ which also satisfies an outgoing radiation condition at ∞. (This condition is incompatible with square integrability, so k^2 is not an eigenvalue.)

Such a solution ψ gives a solution $\psi(x,t)=\exp(-ik^2t)\psi(x)$ of (1.1) and a solution $u(x,t)=\exp(-ikt)\psi(x)$ of (1.2). The approximate lifetimes of these are respectively $(2\kappa\eta)^{-1}$ and η^{-1}. Suppose the perturbation is supported in $\mathscr{B}_R = \{|x|\leqq R\}$. The time spent by an unperturbed particle or wave in $\mathscr{B}_R$ is

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