Commun. Math. Phys. 128, 247–261 (1990)

Communications in Mathematical Physics

A Note on the Global Structure of Supermoduli Spaces*

Gregorio Falqui and Cesare Reina
S.I.S.S.A, International School for Advanced Studies, Strada Costiera 11, I-34014 Trieste, Italy

Abstract. We recall some deformation theory of Susy-curves and study obstructions to projectedness of supermoduli spaces, both from a general standpoint and by means of the local "coordinate charts" most commonly used in the physical literature. We prove that these give rise to a projected atlas for genus $g = 2$ only.

Introduction

Although the burst of interest for string theory in the last years seems to be fading away, there are still interesting open problems to be solved. Among others, the question of the global structure of supermoduli space arises, which is believed to play in superstring theory the rôle of moduli space of algebraic curves in the bosonic model. For instance, when computing amplitudes in superstring theory via a path integral approach, one faces the problem of dealing with odd variables. While the bosonic piece of the Polyakov path integral is well understood as an integral over moduli spaces of algebraic curves, the fermionic part is more embarrassing as the discovery of ambiguities in performing the integration over odd variables pointed out (for a review see e.g. [AMS], [DP] and references quoted therein). To cut a long story short, the basic trouble comes from the fact that in a given supersymmetric gauge the measure for superstrings reduces to a Berezin form, which unluckily is gauge dependent. This is because a supersymmetry transformation induces a small variation of one's gauge choice in a "non-split" way. In other words, the modular parameters change by a nilpotent contribution which in turn induces a change of the string measure by a "total derivative."

Besides these local problems, on which most of the physical literature was focused, there is an even more serious global obstacle to give a mathematically sound definition of the path integral. This may arise from a Gribov-like ambiguity

* Work partially supported by the National Project "Geometria e Fisica," M.P.I.