Commun. Math. Phys. 127, 273–284 (1990)

Communications in Mathematical Physics

On the Derivation of Hawking Radiation Associated with the Formation of a Black Hole

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Abstract. We show how in gravitational collapse the Hawking radiation at large times is precisely related to a scaling limit on the sphere where the star radius crosses the Schwarzschild radius (as long as the back reaction of the radiation on the metric is neglected). For a free quantum field it can be exactly evaluated and the result agrees with Hawking's prediction. For a realistic quantum field theory no evaluation based on general principles seems possible. The outcoming radiation depends on the field theoretical model.

1. Introduction

Classical general relativity leads to the conclusion that very massive stars ultimately end by gravitational collapse, leading at some stage to the formation of a black hole from whose inside no signal can reach an outside observer. Furthermore, for the outside world the black hole has properties of a thermodynamic system in equilibrium ("no hair theorems", entropy) [1, 2]. In an admirable paper [3] Hawking argued that the association of a temperature[1]

$$T = \frac{c^3 \hbar}{8\pi M G} = \frac{a}{2\pi} \hbar / c \tag{1.1}$$

with the black hole surface could be understood by considering quantum field theory in the curved space-time given by the gravitational field of the collapsing star. The simplest model is to take a field obeying linear field equations in a spherically symmetric collapse. At early times, when the star is practically stationary, the state of the quantum field may be assumed to be the "vacuum", i.e. the lowest stationary state in the then static metric. Its development in time or, if one talks in the Heisenberg picture, the expectation values of field quantities at later times can then in principle be calculated by solving the quantum field equations once the gravitational background field is known, and this latter may

[1] M is the mass of the star, a the acceleration at the surface of the black hole, G the gravitational constant