

Braiding Matrices, Modular Transformations and Topological Field Theories in $2+1$ Dimensions

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Abstract. Relations between 3D topological field theories and rational conformal field theories are discussed. In the former framework, we can define the generalized Verlinde operators. Using these operators, we find modular transformations for conformal blocks of one point functions and two point functions on the torus. The result is generalized to higher genus. The correctness of our formulae is illustrated by some examples. We also emphasize the importance of the fusion algebra.

1. Introduction

The classification of conformal field theories (CFT's) is an important issue in recent research [1–24]. There is evidence suggesting that several areas, including CFT's, integrable lattice models, three dimensional topological field theories and the link polynomials, are related by an underlying link, namely, the quantum groups. Witten [26] conjectured that quantum groups should arise naturally in the topological Chern-Simons theories (TCST). Alvarez-Gaumé et al. [22] found that the polynomial equations [15] can be neatly encoded in the structure of a quantum group [32, 33]. However, some important questions remain unanswered. For example, why should quantum group symmetries emerge from the RCFT's only as a secondary phenomenon rather than the first principle in the theory? The profound meaning of the quantum group symmetry can only be understood when such issues are settled.

Further studies on the structure of CFT's may help us catch the key point. In our previous paper we showed that one can express modular transformations in higher genus in terms of braiding matrices. The tool is the TCST. In this paper, we do this explicitly for modular transformations $S(j)$'s for one-point functions on the torus. We find that the $S(j)$'s are not independent data, and that the polynomial equations involving $S(j)$ put further restrictions on braiding matrices. This result

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