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A Topological Characterization of Classical BRST Cohomology

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Abstract. The recent identification of classical BRST cohomology with the "vertical cohomology" of a certain fibration is used to compute it in terms of the classical observables and the topology of the gauge orbits. When the gauge orbits are compact and orientable, a duality theorem is exhibited.

Introduction

The classical BRST cohomology for finite dimensional systems has recently been interpreted geometrically. Let (M, Ω) be a symplectic manifold and $\{\phi_i\}$ a set of first class constraints. Their zero locus, $M_0 = \cap_i \phi_i^{-1}(0)$, is a closed co-isotropic submanifold of M. The hamiltonian vector fields, $\{X_i\}$, associated to the constraints span the null directions of the restriction of Ω to M_0. Since this distributions is involutive, M_0 is foliated by maximal connected submanifolds having the $\{X_i\}$ as their tangent vectors. If this foliation, $\mathscr{F}$, fibers, the space of leaves $\tilde{M}$ can be given a differentiable structure such that the canonical map, $\pi: M_0 \to \tilde{M}$, sending each point in M_0 to the unique leaf it lies in, is a smooth surjection. Moreover $\tilde{M}$ inherits a symplectic structure $\tilde{\Omega}$, making $(\tilde{M}, \tilde{\Omega})$ into a symplectic manifold. The passage from (M, Ω) to $(\tilde{M}, \tilde{\Omega})$ is known as the symplectic reduction of M by M_0.

The tangent bundle of M_0 breaks up as $TM_0 = T\mathscr{F} \oplus N\mathscr{F}$, where $T\mathscr{F} = TM_0^\perp$ is the tangent space to the foliation and $N\mathscr{F}$ is the normal bundle to the foliation. Let $T^*\mathscr{F}$ and $N^*\mathscr{F}$ denote the cotangent and conormal bundles to the foliation, respectively. Under this split, the differential forms, $\Omega(M_0)$, on M_0 decompose as

$$\Omega(M_0) = \bigoplus_{p,q} \Omega^{p,q}(M_0), \tag{1}$$

where $\Omega^{p,q}(M_0)$ is the space of smooth sections through the bundle

$$\wedge^p T^*\mathscr{F} \otimes \wedge^q N^*\mathscr{F}. \tag{2}$$

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