Commun. Math. Phys. 127, 129–144 (1990)

Communications in Mathematical Physics

Q-Analogues of Clifford and Weyl Algebras—Spinor and Oscillator Representations of Quantum Enveloping Algebras

Takahiro Hayashi

Department of Mathematics, Faculty of Science, Nagoya University, Chikusa-ku, Nagoya 464, Japan

Abstract. We introduce q-analogues of Clifford and Weyl algebras. Using these, we construct spinor and oscillator representations of quantum enveloping algebras of type A_{N-1}, B_N, C_N, D_N and $A_{N-1}^{(1)}$. Also we discuss the irreducibility and the unitarity of these representations.

1. Introduction

Clifford and Weyl algebras are two of the most important algebraic objects in theoretical physics. These algebras represent creation and annihilation of particles satisfying Fermi or Bose statistics. Moreover, they have deep connections with many other important algebras, such as Kac–Moody algebras and Virasoro algebras.

Recently, the progress of the quantum inverse scattering method has led to new algebraic structures known as quantum groups. Jimbo [J1] and Drinfeld [D] defined a one-parameter family of Hopf algebras which can be thought of as a q-analogue or quantum deformation of the enveloping algebra of a Kac–Moody algebra (see also [KRS]).

One of the purposes of this paper is to show that the Clifford and Weyl algebras have deformation which is compatible with the deformation of Kac–Moody algebras mentioned above. More precisely, we define q-analogues $\mathscr{A}_q^{\pm}$ of these algebras and construct algebra homomorphisms from quantum enveloping algebras of type A_N, B_N, C_N, D_N and $A_N^{(1)}$ to the algebras $\mathscr{A}_q^{\pm}$. Since $\mathscr{A}_q^{\pm}$ acts on the exterior or polynomial algebra $V^{\pm}$ as usual, we get spinor and oscillator representations of quantum enveloping algebras.

In Sect. 2, we recall the definition of quantum enveloping algebras and its representation theory briefly. In Sect. 3, we define q-analogues $\mathscr{A}_q^{\pm}$ of Clifford and Weyl algebras and study representations $V^{\pm}$ which are irreducible if q is not a root of unity. When q is a root of unity, the representation V^+ of $\mathscr{A}_q^+$ remains irreducible, but the representation V^- of $\mathscr{A}_q^-$ has countably irreducible components. In Sects. 3 and 4, we construct and study the spinor and oscillator representations of quantum enveloping algebras of type A_{N-1}, B_N, C_N and D_N. In