

Z/NZ Conformal Field Theories

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Abstract. We compute the modular properties of the possible genus-one characters of some Rational Conformal Field Theories starting from their fusion rules. We show that the possible choices of S matrices are indexed by some automorphisms of the fusion algebra. We also classify the modular invariant partition functions of these theories. This gives the complete list of modular invariant partition functions of Rational Conformal Field Theories with respect to the $A_N^{(1)}$ level one algebra.

Introduction

Since the important work of Belavin, Polyakov and Zamolodchikov [1], it has become clear that Conformal Field Theory (CFT) has exceptional properties in two dimensions. In particular, the symmetry of these theories is so huge that the hope of a possible classification has emerged. The main idea is to classify the states of a CFT using representations of the symmetry algebra of the theory.

Rational Conformal Field Theories, where only a finite number of representations appear, provide the simplest situation. Recently the application of these ideas through a detailed study of modular invariance on any surface (punctured or not punctured and in any genus) has produced remarkable results [2, 3, 4, 5]. One of the most amazing facts is the appearance of an unexpected relationship between the fusion rules of a RCFT and the modular properties of genus-one characters $\text{tr}(q^{L_0})$. This relationship, due to Verlinde [6], leads to a new method of investigation of RCFTs: one can try to classify RCFTs starting from their fusion rules. This paper is an attempt at giving a few illustrations of this method.

In this approach, the first step is of course to find some fusion rules to work with. It appears that (finite) group theory provides us such material quite spontaneously. Verlinde's work enables us to recover the modular properties of

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