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A Perturbed Mean Field Model of An Interacting Boson Gas and the Large Deviation Principle

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Abstract. This is a study of the equilibrium thermodynamics of a mean-field model of an interacting boson gas perturbed by a term quadratic in the occupation numbers of the free-gas energy-levels. We prove the existence of the pressure in the thermodynamic limit. We obtain also a variational formula for the pressure; this enables us to compare the effect of a smooth quadratic perturbation with that of a singular one (the Huang–Yang–Luttinger model). The proof uses a large deviation result for the occupation measure of the free boson gas which is of independent interest.

1. Introduction

The rigorous investigation of the thermodynamics of a system of bosons based on the full quantum mechanical hamiltonian and using a realistic pair- potential seems beyond the reach of present methods. Either one must use a very special pair-potential or one must truncate the hamiltonian. The first course was followed by Lieb and Liniger [1] who diagonalized the full hamiltonian for a boson gas with a δ-function pair-potential in one dimension using what is now known as the Bethe Ansatz. Using the results of [1], Yang and Yang [2] gave a formula for the pressure in this model; in a recent paper [3], we proved that if the Bethe Ansatz wave-functions form a complete set then the grand canonical pressure is given by the Yang–Yang formula. The proof in [3] uses probabilistic methods (Varadhan's Laplacian asymptotics [4] based on the large deviation principle).

The second course has been followed by many authors. One such approach is to use a hamiltonian which is a function of the free-gas occupation numbers; the reader is referred to the book of Thouless [5] for an introduction to these models. In this paper, we continue our investigation by probabilistic methods of such models: we study a smooth perturbation of a mean-field model of interacting bosons (for perturbations of mean-field models in other contexts, see Bricmont and Fontaine [6]).

In [7], we studied the equilibrium thermodynamics of the Huang–Yang–Luttinger model [12] of a boson gas with a hard-sphere repulsion using large