Commun. Math. Phys. 126, 507–533 (1990)

Communications in Mathematical Physics

A System of One Dimensional Balls with Gravity

Maciej P. Wojtkowski*

Department of Mathematics, University of Arizona, Tucson, AZ 85721, USA

Abstract. We introduce a Hamiltonian system with many degrees of freedom for which the nonvanishing of (some) Lyapunov exponents almost everywhere can be established analytically.

Introduction

Dynamical properties of Hamiltonian systems with many degrees of freedom are reasonably well understood. At least we think we know what to expect. The two major ingredients of the dynamics are quasiperiodic motions and a component with strong mixing properties. The other features of the dynamics like Smale's horseshoes and Cantori of Mather and Aubry take place on sets of measure zero and probably do not matter that much in the case of many degrees of freedom.

There are Hamiltonian systems where the whole phase space is filled with quasiperiodic motions. These are so-called completely integrable systems and there is an ample supply of examples. By the KAM theory nonresonant quasiperiodic motions survive small perturbations. For many degrees of freedom the smallness of the perturbation is probably very restrictive so that for a typical system only a small portion of the phase space is filled with quasiperiodic motions. Nonetheless their actual or possible presence thwarts attempts to study the mixing component since the two components have to be intricately intertwined. So far only crude models were found where the two types of behavior were shown to coexist ([W1, Pr1, W2, Del]) but the success there depends on a simple splitting of the phase space which is destroyed under any kind of perturbation. Thus for the case of mixed behavior we have virtually no examples and no theorems about the mixing component. We have though a solid conjecture: for a "typical" Hamiltonian system there is a component of positive measure where Lyapunov exponents are nonzero.

The systems which do not have the quasiperiodic component and where the strong mixing is present in all of the phase space should be more accessible, yet

* Supported in part by the Sloan Foundation and the NSF Grant DMS-8807077