Commun. Math. Phys. 126, 483–506 (1990)

Communications in Mathematical Physics

First Order Phase Transitions in Unbounded Spin Systems. II. Completeness of the Phase Diagram

Christian Borgs[1] and Roger Waxler[2]

1 Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland

2 Dept. of Mathematics, University of British Columbia, Vancouver, BC, Canada

Abstract. We continue our analysis of unbounded spin systems with nearest neighbor interaction W and a single spin potential V which has N deep and widely separated minima. In this second part we show that all translation invariant phases obeying a certain regularity condition are convex combinations of the stable phases determined in the first part of this paper. For periodic boundary conditions each stable phase contributes with the same weight in the infinite volume limit.

1. Introduction

In the first part [1] of this work we have shown that for an unbounded spin model with Hamiltonian

$$H = \sum_x V(R_x) + \sum_{\langle xy \rangle} W(R_x, R_y), \tag{1.1}$$

where V has N deep and widely separated minima and W is a kinetic energy type interaction (see Sect. 1 of [1] for the precise assumptions), the stable phases[1] are characterized by the condition that the free energy, h'_q, of a certain truncated model is minimal. As we have seen this is enough to construct translation invariant states, $\langle \cdot \rangle_q$, which are small perturbations of the corresponding Gaussian approximations. Furthermore these states show exponential clustering and hence are, in the usual langauge of statistical mechanics, pure states of the sytem (1.1).

The goal of this paper is to show that these are in fact *all* pure states, i.e. that any translation invariant equilibrium state, $\langle \cdot \rangle$, is a convex combination of the stable states, $\langle \cdot \rangle_q$, constructed in [1]. To be more precise assume that V and W obey assumption A.0 through A.3 of [1] (guaranteeing the convergence of the cluster expansions developed in [1] and hence the existence of $\langle \cdot \rangle_q$). Assume in addition the following

---

[1] Throughout this paper we will use letters q, q', etc. to denote stable phases, while the letters m, m', etc. denote arbitrary (stable or unstable) phases