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Uniqueness of the Translationally Invariant Ground State in Quantum Spin Systems

T. Matsui*

Department of Mathematics and Computer Science, University College of Swansea, Singleton Park, Swansea SA2 8PP, Wales, UK

Abstract. We introduce a class of quantum spin systems on $\mathbb{Z}^d$. We show that the translationally invariant ground state is unique for this system if it is in a strong external field.

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In this paper, we study translationally invariant ground states of quantum spin systems. We consider our problems in C^* algebraic framework. (See [2].) We introduce a class of quantum spin Hamiltonians which are translationally invariant and have the Perron Frobenius property. The Hamiltonians contain an external field term. We establish uniqueness of translationally invariant ground state if the strength of the external field is sufficiently large. Our class of Hamiltonian contains Quantum Ising models and Heisenberg models. In the due course of the proof, we will see that the invariant ground states of our systems can be realized by (finite temperature) Gibbs states of classical spin models. This correspondence has been first found by Kirkwood and Thomas in [4].

To be more precise, we introduce some notations. Let $\sigma_\alpha(\alpha = x, y, z)$ be Pauli spin matrices,

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{1.1}$$

We consider the C^* algebra $\mathscr{A} = \bigotimes_{\mathbb{Z}^d} M_2(\mathbb{C})$ and by $\sigma_\alpha^{(j)}(j \in \mathbb{Z}^d, \alpha = x, y, z)$ we denote the Pauli spin matrix on site j. Hence $\sigma_\alpha^{(j)}$ satisfies the following commutation relations:

$$[\sigma_\alpha^{(j)}, \sigma_\beta^{(k)}] = \sigma_\alpha^{(j)}\sigma_\beta^{(k)} - \sigma_\beta^{(k)}\sigma_\alpha^{(j)} = 0, \quad \text{if} \quad k \neq j, \tag{1.2a}$$

$$\sigma_\alpha^{(j)}\sigma_\beta^{(j)} = i\varepsilon_{\alpha\beta\gamma}\sigma_\gamma^{(j)}, \tag{1.2b}$$

where $\varepsilon_{\alpha\beta\gamma}$ is the totally antisymmetric tensor with $\varepsilon_{xyz} = 1$.

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