Commun. Math. Phys. 126, 433–452 (1990)

Communications in Mathematical Physics

Projective Embeddings of Complex Supermanifolds

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Abstract. We generalize the Kodaira Embedding Theorem and Chow's Theorem to the context of families of complex supermanifolds. In particular, we show that every family of super Riemann surfaces is a family of projective superalgebraic varieties.

1. Introduction

In the past few years there has been a great deal of mathematical activity concerning supermanifolds, both real and complex. While much of this work may seem unrelated to the physical motivations of the field, it must be remembered that the subject came into being when physicists realized that it is perfectly consistent to introduce spaces with anti-commuting coordinates [2]; their original train of thought was inextricably linked with the Fermi statistics of quantum field theory, and soon resulted in the formulation of supersymmetric field theories (e.g. [16, 20, 26]). It is therefore perhaps not surprising that the deformation theory of complex supermanifolds has now, with the growing prominence of superstring theory [9, 12], proved to be germane to current physics.

The calculation of amplitudes in superstring theory [9] is supposed to involve integration over the moduli space [6, 15] of super Riemann surfaces. Unfortunately, this is a rather unruly object, and, in particular, its non-compactness tends to make such integrals ill-defined [23]. On the other hand Mumford [18] has given us a beautiful compactification of the moduli space of Riemann surfaces by treating them as algebraic curves, and Deligne [4] has announced that the same can be done for super Riemann surfaces. In order to carry out such a program, it is first necessary to show that families of complex supermanifolds of dimension $1|1$ can always be thought of as families of projective superalgebraic varieties. In this paper,

* Research supported in part by NSF grant DMS-8704401

** Research supported in part by NSF grant DMS-4253943

*** Research also supported in part by NSF grant DMS-4253943