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Meromorphic Extensions of Regular Holonomic Distributions

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Abstract. In this paper we consider distributions of the form $1/P$ and $\int dy/P(x, y)$, where P is a polynomial. Using results by Kashiwara and Kawai we give fairly accessible proofs that these expressions can be defined as regular holonomic distributions by utilising a meromorphic parameter. We also discuss distributions of the form μ/P and their direct image $\int \frac{\mu dy}{P(x, y)}$, when one knows that μ is a regular holonomic distribution. All these distributions are relevant to the study of renormalised Feynman integrals.

1. Introduction

In [1] Kashiwara and Kawai develop a micro-local theory of holonomic systems with regular singularities. This theory is then applied to show that an analytically renormalised Feynman function satisfies a holonomic system of linear partial differential equations with regular singularities, and as a by-product of their general theory they show that the Feynman functions are all Nilsson class functions [2] (see also [3]). They have also proved that the system of differential equations corresponding to a Feynman function has a Lagrangian characteristic set which is contained in the extended Landau variety in $T^*(\mathbf{C}^N)$, for some integer N. This implies that the wave front set of the Feynman function (also for complex momenta) is contained in the extended Landau variety. We refer to [4, 5] for further discussion about the relevance of regular holonomicity to physics. In this paper we will show that a similar class of expressions, given by rational functions with singularities, also can be defined as regular holonomic distributions. Indeed, it follows from these results that the analytically renormalised Feynman integrals are regular holonomic (see the remark in Sect. 4). We do not claim essential originality because some of our main results are implicitly contained in the work [6]. However, the algebraic theory of regular holonomic $\mathscr{D}$-modules has its special features and is for example presented in [7]. Here we offer reasonably self-contained proofs concerned with existence of meromorphic extensions of certain tempered distributions, which