Commun. Math. Phys. 126, 103–131 (1989)

Communications in
Mathematical
Physics

Discontinuity of the Wilson String Tension in the 4-Dimensional Lattice Pure Gauge Potts Model

L. Laanait*, A. Messager, and J. Ruiz

Centre de Physique Théorique**, CNRS-Luminy, Case 907, F-13288 Marseille Cedex 9, France

Abstract. We consider the 4-dimensional q-state pure gauge Potts model. For q large enough, we give a new proof of the existence of a unique coupling constant β_t, where a first order phase transition occurs. Moreover we prove the following new results: The string tension is discontinuous at β_t, the Wilson parameter exhibits at β_t a direct transition from an area law decay (quark confinement) to a perimeter law decay (quark deconfinement).

1. Introduction

The q-state Potts lattice gauge model was introduced by Kogut [8], according to Wilson's formulation of gauge theories on a lattice [9]. This model was considered as a natural extension of the Ising lattice gauge model, treated by several authors (see [10] and references therein). Still now there exist a great deal of rigorous results on the deconfining phase transition for the pure gauge Ising model in dimension $d \geqq 3$ [11, 12] and for the $U(1)$ model defined with the Wilson's action in [14–16]. Also Monte-Carlo experiments were proposed for the Ising gauge model in [17, 18].

Concerning the 4-dimensional Potts pure gauge model there were some previous results on its phase diagram in [8]. An improvement of the results obtained in [8] was suggested in [19] by using a perturbative cluster expansion. However topological problems inherent to the 4-d Potts gauge model were not elucidated, this was emphasized by Aizenman and Fröhlich in [20].

In both its scalar and gauge formulations the Potts model gives rise to great investigations. For q large enough it exhibits a first order phase transition either in dimension $d \geqq 2$ for the scalar model or for the gauge model in dimension $d \geqq 3$. Namely, there exists a transition point where ordered phases coexist with a "disordered" one as proved in [1] by using Reflexion positivity [2]. Later a

---

* Université d'Aix-Marseille II, Faculté des Sciences de Luminy (On leave from Ecole Normale Supérieure, Rabat, Morocco)

** Laboratoire propre LP. 7061, Centre National de la Recherche Scientifique