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Surfaces and Peierls Contours: 3-*d* Wetting and 2-*d* Ising Percolation

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Dedicated to Roland Dobrushin

Abstract. A natural model of a discrete random surface lying above a two-dimensional substrate is presented and analyzed. An identification of the "level curves" of the surface with the Peierls contours of Ising spin configurations leads to an exactly solvable free energy, with logarithmically divergent specific heat. The thermodynamic critical point is shown to be a wetting transition at which the surface height diverges. This is so even though the surface has no "downward fingers" and hence no "entropic repulsion" from the substrate.

I. Introduction

Simple geometrical models of surfaces can be useful for understanding various aspects of interface structure and phase transitions. In this paper we analyze in detail a particular three-dimensional random surface model, first introduced in [AN] (see Sect. II for the definition). The main conclusions are:

(i) The model is directly related to the standard 2-*d* Ising ferromagnet and thus has an exactly solvable free energy with a logarithmically divergent specific heat singularity.
(ii) The singularity corresponds to a transition from partial to complete wetting; i.e., from finite to infinite height of the surface above the substrate.

Elementary models of surfaces can be constructed on the simple cubic lattice by assigning a single-valued variable $h(x,y)\in\mathbb{Z}$ to each point $(x,y)\in\mathbb{Z}^2$. This variable is the height of the surface; we place a unit-sided square, or plaquette, symmetrically through the point $(x,y,h(x,y))$ parallel to the (x,y) plane. We then fill in between these plaquettes with others whose normals are parallel to the $\mathbb{Z}^2$ plane. Such a model, with configurational energy,

$$J\sum_{|\mathbf{r}-\mathbf{s}|=1}|h(\mathbf{r})-h(\mathbf{s})|^p,$$

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