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The Coulomb Gas at Low Temperature and Low Density*

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Dedicated to Roland Dobrushin

Abstract. We study the quantum Coulomb Gas of N particles with Hamiltonian H at low temperature and negative values of the chemical potential μ. If μ is sufficiently negative the Coulomb gas is approximately a perfect rare gas of charged particles, as expected. The interesting fact is that for higher (but still negative) values of μ the gas changes to a rare gas of some atom or molecule (which is most likely neutral). The type of molecule is determined by the ground state of the Hamiltonian $H-\mu N$ with center of mass motion removed.

Introduction

In this paper we are concerned with the thermodynamic properties of a quantum mechanical Coulomb gas of nuclei and electrons at very low temperature and density and to validate certain predictions of the Saha equation [10, 12], which is the equation that governs the regime. (Actually, Saha was interested in the solar chromosphere where the temperature is high by earthly standards and the pressure is low, but this equation extends to the regime of very low temperature and even lower density that we are considering here.)

Consider a system composed of S species of charged particles (electrons and various nuclei) placed in a large box Λ of volume $|\Lambda|$. If N_i is the number of particles of species i, then $\varrho_i = N_i/|\Lambda|$ is its density. If the ϱ_i's and $\beta = (k_B T)^{-1}$ (with k_B = Boltzmann's constant and T = temperature) are fixed and $\Lambda \to \infty$ in a suitable way, we expect that, the long-range nature of the Coulomb potential notwithstanding, the intensive quantities such as pressure, free energy per unit volume, etc. have well defined limits and that these limits should have the correct convexity

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