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An Index Theorem for Super Derivations*

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Dedicated to Roland Dobrushin

Abstract. We show that the Chern character given by a super-KMS functional on a quantum algebra can be interpreted in terms of the index of a super derivation on a projection of the algebra.

I. Introduction

The heat kernel representation of the index of a Fredholm operator provides a natural connection between statistical physics (through the partition function) and geometry (regarding the index as a topological invariant). The equivalence in certain statistical mechanics models between the Gibbs variational principles (as expressed through the equation of Dobrushin, Lanford and Ruelle) and the KMS condition provided a fundamental interpretation of trace invariants for operator algebras. Recently the importance of the KMS property (in a super or graded setting) has emerged as a fundamental starting point for the definition of a class of geometric invariants. This subject unifies Connes' noncommutative differential geometry, analysis in an infinite dimensional setting, and ideas from statistical and particle physics. Thus it is especially appropriate to dedicate this note to Roland Dobrushin.

Our purpose here is to investigate an aspect of index theory for a super derivation without assuming that it is generated by a Fredholm operator, without assuming compactness or a bound on the dimension of an underlying manifold, and without necessarily obtaining integral invariants. In place of the standard assumptions we suppose that we are given a super-KMS functional on a quantum algebra $\mathscr{A}$ (defined below). This elementary assumption leads to a Chern character τ for the quantum algebra, and a pairing between τ and the K_0 group of the even part of the algebra. We show here that this pairing can be interpreted in terms of the

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