Commun. Math. Phys. 125, 71–79 (1989)

Communications in Mathematical Physics

From PCA's to Equilibrium Systems and Back*

Sheldon Goldstein, Roelof Kuik**, Joel L. Lebowitz, and Christian Maes***
Department of Mathematics and Physics, Rutgers University, New Brunswick, NJ 08903, USA

Dedicated to Roland Dobrushin

Abstract. Stationary measures for probabilistic cellular automata (PCA's) in d dimensions give rise to space-time histories whose statistics may naturally be described by Gibbs states in $d+1$ dimensions for an interaction energy $\mathscr{H}$ obtained from the PCA. In this note we study the converse question: Do all Gibbs states for this $\mathscr{H}$ correspond to statistical space-time histories for the PCA? Our main result states that the answer is yes, at least for translation invariant or periodic Gibbs states. Thus ergodicity questions for PCA's can, at least partially, be formulated as questions of uniqueness of Gibbs states.

1. Introduction

Probabilistic Cellular Automata (PCA's) – which include the deterministic ones as special cases – are interesting subjects of study both mathematically and as models of various phenomena [1–6]. In this note we study some aspects of the connection between PCA's in d dimensions and statistical mechanical Gibbs states in $(d+1)$ dimensions [7, 8], the extra dimension being the discrete time. Our objective is to facilitate the transfer of information, particularly rigorous results, between these subjects.

The state space of our PCA is the set of spin configurations $\underline{\sigma}$ on a d-dimensional lattice, at each site of which there is a spin variable which takes on a discrete set of values. We shall consider for concreteness the lattice $\mathbb{Z}^d$ and Ising spin variables taking on the values of ± 1. The PCA describes a stochastic discrete time evolution of the spin configuration on $\mathbb{Z}^d$. We denote the value of the spin at site $i \in \mathbb{Z}^d$ at time $n \in \mathbb{Z}$ by $\sigma_{n,i} = \pm 1$, and write $\underline{\sigma}_n = \{\sigma_{n,i}\}$ for the spin configuration at time n.

* Research supported by NSF Grants DMR-86-12369 and DMS-85-12505
** Permanent address: Erasmus University, Faculteit Bedijfskunde Rotterdam
*** Aspirant N.F.W.O., Instituut voor Theoretische Fysika, K.U. Leuven, Celestijnenlaan 200D, B-3030 Leuven, Belgium