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Stretched Exponential Decay in a Kinetic Ising Model with Dynamical Constraint

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Dedicated to Roland Dobrushin

Abstract. We show that for the standard nearest neighbor spin-flip dynamics in one dimension with the constraint of constant energy the spin-spin correlation function decays as $\exp[-c\sqrt{t}]$ for large t. We prove an upper and lower bound. The coefficient c of the lower bound is given as the solution of a variational problem and is conjectured to be exact.

1. Introduction

Experimentally it is found that in many materials the decay to equilibrium is not exponentially fast but better fitted by a stretched exponential of the form $\exp[-(t/\tau)^\beta]$, $0<\beta<1$. This is known as the Kohlrausch-Williams-Watts law. In a well known paper Palmer, Stein, Abrahams, and Anderson [1] tried to find a general, material-independent explanation. They argued that in systems consisting of many components which decay in parallel the decay has to be exponential, provided of course each component individually relaxes exponentially. However if there are dynamical constraints, which they imagine to be of a hierarchical nature, then stretched exponential decay is likely to occur.

Presumably the simplest model for parallel decay are independent spin-flips. Each component has then only two possible states and flips between them at random times. This yields exponential decay on the average. We are interested in how the relaxation is modified when a *local* dynamical constaint is imposed. Such models have been used in order to understand properties of glassy dynamics and of the glass transition [2, 3]. We study here the one-dimensional case only. It is physically of its own interest as describing relaxation in solid amorphous polymers [4, 5].

Let us consider then a one-dimensional spin flip process. The spin variable $\sigma(x)$ at site x takes the values ± 1. The state space is therefore $\{-1,1\}^{\mathbb{Z}}$. A spin configuration is denoted by σ. σ^x is the configuration σ with $\sigma(x)$ replaced by $-\sigma(x)$ ($=$spin flip at x). We make a specific choice of nearest neighbor flip rates, namely

$$\begin{aligned} c(x,\sigma)=&\tfrac{1}{2}(1-\sigma(x-1)\sigma(x+1))+\kappa(1+e^{2\beta})(1+\sigma(x-1)\sigma(x+1))\\ &+\kappa(1-e^{2\beta})\sigma(x)(\sigma(x-1)+\sigma(x+1)), \end{aligned} \tag{1.1}$$