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Braid Matrices and Structure Constants for Minimal Conformal Models

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Abstract. Using the Feigin–Fuchs representation of minimal conformal models in a form introduced recently by one of us, the braid group representation matrices, describing the analytic continuation properties of conformal blocks, are computed. In a suitable normalization, their matrix elements are shown to essentially factorize into pairs of Boltzmann weights of critical RSOS models in a certain limit of the spectral parameter. These Boltzmann weights are related to quantum group R-matrices by the vertex-SOS transformation. We show that the crossing symmetry of the four-point function in left-right symmetric models follows from a quantum group relation, also called crossing symmetry. This observation gives a simple way to evaluate the structure constants.

1. Introduction

In recent times, much study was devoted to the connection between conformal field theory and representations of the braid group [1–11]. The reasons for this interest are two-fold: First, understanding the braid group representation carried by the conformal blocks is necessary to complete the conformal bootstrap program [12] in the general case. Second, this method, possibly in connection with modular invariance, might ultimately lead to a classification of two-dimensional conformal theories, at least of rational theories.

Some time ago, Tsuchiya and Kanie [1] found a connection between the braid matrices describing the monodromy of conformal blocks of the fundamental field in the $SU(2)$ WZW model and the Temperley–Lieb–Jones algebra. The same structure was seen to arise [4] for the braid matrices of the $\phi_{(12)}$ field in minimal models [12]. These matrices are connected by the vertex-SOS transformation to the spin $\frac{1}{2}$ R-matrices of exactly solvable vertex models in a certain limit of the parameters. By the fusion procedure, R-matrices corresponding to higher spin

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