Commun. Math. Phys. 124, 629–645 (1989)

Communications in Mathematical Physics

Functional Determinants on Mandelstam Diagrams*

Eric D'Hoker[1] and D. H. Phong[2]

[1] Department of Physics, University of California, Los Angeles, CA 90024, USA
[2] Department of Mathematics, Columbia University, New York, NY 10027, USA

Abstract. We investigate the special properties of Mandelstam metrics in regard to changing weights in path integrals and relations between determinants of different spins. Regularizations of determinants are discussed along the lines of Sonoda. Weyl anomalies developing at zeroes of metrics in reparametrization invariant regularizations are evaluated in terms of Arakelov metrics. Holomorphic forms are constructed, and determinant identities for Arakelov and Mandelstam metrics rigorously established for any weight and generic even and odd spin structures.

1. Introduction

Determinants of Laplacians are playing an increasingly important role in diverse areas of physics and geometry. The foundations of their theory have however been developed mostly for compact manifolds and regular metrics. In this paper we wish to examine properties of determinants on certain surfaces with degenerate metrics and punctures which arise as Feynman diagrams in string theory.

It is a fundamental principle of string theory that scattering amplitudes depend only on the conformal class of the Feynman diagrams and not on particular choices of metrics within the class. Nevertheless it is sometimes useful to select a privileged representative to carry out explicit computations. For surfaces, two metrics arise which in some sense are opposites of one another: metrics with constant curvature, and metrics with all curvature concentrated at isolated points. The former are familiar from hyperbolic geometry, and their determinants now well understood in the compact case, as special values of the Selberg zeta function [1]. Examples of the latter are $|v_+|^4$ or $|\omega_z|^2$, where v_+ and ω_z are respectively a meromorphic spinor [2] and a meromorphic form [3, 4]. Their poles can be viewed as punctures on the surface indicating external string states, and their zeroes as interaction

* Work supported by the National Science Foundation under grants PHY-80-19754 PHY-86-13201 and DMS-87-04209