

# Chern Numbers, Quaternions, and Berry's Phases in Fermi Systems

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**Abstract.** Yes, but some parts are reasonably concrete.

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