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A Weyl Group for the Virasoro and $N=1$ Super-Virasoro Algebras

V. K. Dobrev*

International Centre for Theoretical Physics, Trieste, Italy

Abstract. We introduce a Weyl group for the highest weight modules over the Virasoro algebra and the Neveu-Schwarz and Ramond superalgebras. Using this group we rewrite the character formulae for the irreducible highest weight modules over these algebras in the form of the classical Weyl character formula for the finite-dimensional irreducible representations of semi-simple Lie algebras (and also of the Weyl-Kac character formula for the integrable highest weight modules over affine Kac-Moody algebras). This is the same group we introduced recently in order to rewrite in a similar manner the characters of the singular highest weight modules over the affine Kac-Moody algebra $A_1^{(1)}$.

Introduction

The present paper is an attempt to extend the notion of Weyl group to the Virasoro and $N=1$ super-Virasoro algebras. As we know Weyl groups and their generalizations are very important in the representation theory of semi-simple Lie algebras $\mathfrak{G}_0$, of affine Kac-Moody algebras $\mathfrak{G}$ and of super-algebras $\mathfrak{G}_S$ associated with (generalized) Cartan matrices (see e.g. [1–5]). In the generic cases the Weyl(-Kac) group is essential in the formulae describing the characters of the irreducible finite-dimensional representations of $\mathfrak{G}_0$, of the integrable representations of $\mathfrak{G}$ and of the typical representations of $\mathfrak{G}_S$ [1–5]. Nevertheless, there are large and interesting classes of representations for which the Weyl(-Kac) group does not play a similar role. Examples of these are the singular representations of affine Kac-Moody algebras [6, 7] characterized by the fact that the central charge is equal to the dual coxeter number and the atypical representations of finite-dimensional $\mathfrak{G}_S$. Recently [8], using results of [9] we derived character formulae for singular highest weight modules over $A_1^{(1)}$. Then we introduced new Weyl groups W_a, W_a^+, which we used to rewrite these character formulae so that they look exactly as the usual Weyl(-Kac) formulae with the Weyl(-Kac) group W replaced by W_a or W_a^+.

* Permanent address: Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 72, Boul. Lenin, BG-1784 Sofia, Bulgaria