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On The Structure of Unitary Conformal Field Theory. I. Existence of Conformal Blocks

G. Felder*, J. Fröhlich and G. Keller
Theoretical Physics, ETH-Hönggerberg, CH—8093 Zürich, Switzerland

Abstract. We study the general mathematical structure of unitary rational conformal field theories in two dimensions, starting from the Euclidean Green functions of the scaling fields. We show that, under certain assumptions, the scaling fields of such theories can be written as sums of products of chiral fields. The chiral fields satisfy an algebra whose structure constants are the matrix elements of Yang–Baxter- or braid-matrices whose properties we analyze. The upshot of our analysis is that two-dimensional conformal field theories of the type considered in this paper appear to be constructible from the representation theory of a pair of chiral algebras.

1. Introduction

In this paper we study the general structure of unitary rational conformal field theories in two dimensions. The starting point of our analysis is motivated by concepts of two-dimensional statistical mechanics: The basic properties of a statistical system are coded into its thermodynamic and correlation functions. The correlation functions are expectations of products of local order- and disorder variables in a Gibbs equilibrium state. If the system is at a critical point its correlation functions tend to exhibit asymptotic Euclidean- and scale invariance, as one learns from the study of exactly solved models and the renormalization group. Scaling limits of the correlation functions then exist. They turn out to be the Euclidean Green functions of some Euclidean field theory. If the underlying statistical system has a self-adjoint transfer matrix, the scaling limits of its correlation functions satisfy reflection positivity. A variant of Osterwalder–Schrader reconstruction then permits us to associate with the sequence of scaling limits of correlation functions of such a system a unitary relativistic quantum field theory. At a critical point the scaling limits of correlation functions of scaling operators are Möbius-invariant. This invariance property, combined with reflection

* Temporary address: The Institute for Advanced Study, Princeton, New Jersey 08540, USA