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The Yang–Yang Thermodynamic Formalism and Large Deviations

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Abstract. The partition function for a one-dimensional system of Bosons with repulsive delta-function interaction is investigated. We prove that if the Bethe Ansatz eigenfunctions form a complete set then the grand canonical pressure is given by the Yang–Yang formula. The proof uses a probabilistic formalism to express the partition function as an expectation with respect to a probability measure on a Banach space of measures; the asymptotic behaviour of the expectation in the thermodynamic limit is determined by the Large Deviation Principle. This method is applicable in situations in which the Hamiltonian can be diagonalised using the Bethe Ansatz.

1. Introduction

Often, in mathematical physics, we are faced with the problem of determining the asymptotic behaviour, for large l, of a sequence

$$\{\text{trace}\exp[-\beta\mathscr{H}^l]|l=1,2,\dots\},$$

where β is a positive real number and $\{\mathscr{H}^l|l=1,2,\dots\}$ is a sequence of self-adjoint operators on some Hilbert space. The problem arises, for example, in many-body theory; here $\mathscr{H}^l$ is the Hamiltonian of the system, β is the inverse temperature and the volume V_l of the system increases as l increases. In this setting, there are not many cases in which the problem has been solved. For a long time, only for the free quantum gases, boson and fermion, was an explicit expression known for

$$\lim_{l\to\infty}\frac{1}{\beta V_l}\ln\text{trace}\exp[-\beta\mathscr{H}^l].$$

In 1969, Yang and Yang [1] made a notable advance: they developed a thermodynamic formalism for dealing with those interacting systems whose Hamiltonians can be diagonalized with the help of the Bethe Ansatz. Yang and Yang [1] applied their formalism to the quantum non-linear Schroedinger model whose Hamiltonian had been diagonalized six years previously by Lieb and Liniger [2]. In recent years, as more and more problems have succumbed to the Bethe