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Cohomology of Power Sets with Applications in Quantum Probability

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Abstract. Square integrable Wiener functionals may be represented as sums of multiple Itô integrals. This leads to an identification of such functionals with square integrable functions on the symmetric measure space of the Lebesgue space R_+. When the pointwise product of Wiener functionals is thus carried over, the product takes a pleasing form (cf. Wick's theorem) and various non-commutative perturbations of this "Wiener product" have been considered. Here we employ cohomological arguments to analyse deformations of an abstract Wiener product. This leads to the construction of Lévy fields which are neither bosonic nor fermionic, and also gives rise to homotopies between quasi-free boson and fermion fields. Finally we unify existence and uniqueness results for quantum stochastic differential equations by treating mixed noise differential equations.

Introduction

Any square integrable Wiener functional F has an expansion in terms of multiple Itô integrals:

$$F = f_0 + \sum_{n=1}^{\infty} \int \cdots \int_{\nabla^n} f_n(t_1, \dots, t_n) \mathrm{dB}_{t_1} \cdots \mathrm{dB}_{t_n},$$

where ∇^n is the increasing quadrant $\{\mathbf{t} \in \mathbb{R}_+^n : t_1 < \cdots < t_n\}$. The sequence $\{f_n\}$ may be viewed as a single function f on the collection $\Gamma(\mathbb{R}_+)$ of all subsets of $\mathbb{R}_+$ having finite cardinality:

$$f(\emptyset) = f_0; \quad f(\sigma) = f_n(\mathbf{s}) \quad \text{for} \quad n = \#\sigma \geqq 1,$$

where $s_1, s_2, \dots, s_n$ is an enumeration of the set σ in increasing order. There is a natural measure λ on $\Gamma(\mathbb{R}_+)$, derived from Lebesgue measure on $\mathbb{R}_+$, for which the correspondence $F \to f$ is an isometric isomorphism from $\mathscr{W}$, the space of square integrable Wiener functionals, to $\mathscr{F} = L^2(\Gamma(\mathbb{R}_+), \lambda)$. Under pointwise

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