

New Weyl Groups for $A_1^{(1)}$ and Characters of Singular Highest Weight Modules

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Abstract. We consider singular Verma modules over $A_1^{(1)}$, i.e., Verma modules for which the central charge is equal to minus the dual Coxeter number. We calculate the characters of certain factor modules of these Verma modules. In one class of cases we are able to prove that these factor modules are actually the irreducible highest modules for those highest weights. We introduce new Weyl groups which are infinitely generated abelian groups and are proper subgroups or isomorphic between themselves. Using these Weyl groups we can rewrite the character formulae obtained in the paper in the form of the classical Weyl character formula for the finite-dimensional irreducible representations of semisimple Lie algebras (respectively Weyl-Kac character formula for the integrable highest weight modules over affine Kac-Moody algebras) so that the new Weyl groups play the role of the usual Weyl group (respectively affine Weyl group).

0. Introduction

The notion of a Weyl group is very essential for the representation theory of semisimple Lie algebras and groups. It allows the nice classical formula of Weyl for the characters of the finite-dimensional irreducible representations L of the semisimple Lie algebras. Connectedly it permutes the weights of the finite-dimensional irreducible representations L of the semisimple Lie algebras and determines the embedding pattern of reducible Verma modules over such algebras. Later the notion of a Weyl group was generalized for affine Kac-Moody algebras [1] and for finite-dimensional Lie superalgebras [2]. For affine Kac-Moody algebras the Weyl character formula holds for the integrable highest weight modules L by replacing the Weyl group with the affine Weyl group [3]. The affine Weyl group or Weyl-Kac group determines the embedding pattern of Verma modules [4, 5] except for the so-called *singular Verma modules*. The latter were introduced in [6] by the

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