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An Analogue of P.B.W. Theorem and the Universal *R*-Matrix for $U_h sl(N+1)$

Marc Rosso
Centre de Mathematiques, Ecole Polytechnique, F-91128, Palaiseau Cedex, France

Abstract. One uses Drinfeld's quantum double construction and a basis à la Poincaré–Birkhoff–Witt in $U_h n_+$ to compute an explicit formula for the quantum *R*-matrix.

0. Introduction

1. Definition: [1,2] $U_h sl(N+1)$ is the topologically free $C[[h]]$ algebra generated by $X_i, Y_i, H_i, 1 \leqq i \leqq N$, with the relations:

$$[H_i, H_j] = 0, \quad [H_i, X_j] = \alpha_j(H_i)X_j,$$

$$[H_i, Y_j] = -\alpha_j(H_i)Y_j, \quad 1 \leqq i, j \leqq N,$$

$$[X_i, Y_j] = \delta_{ij}\frac{\mathrm{sh}\left(\frac{h}{2}H_i\right)}{\mathrm{sh}\left(\frac{h}{2}\right)},$$

for $|i-j| = 1, X_i^2 X_j - (e^{h/2} + e^{-h/2})X_i X_j X_i + X_j X_i^2 = 0,$

$$Y_i^2 Y_j - (e^{h/2} + e^{-h/2})Y_i Y_j Y_i + Y_j Y_i^2 = 0.$$

It is a Hopf algebra for the coproduct Δ:

$$\Delta(H_i) = H_i \otimes 1 + 1 \otimes H_i, \quad \Delta(X_i) = X_i \otimes \exp\left(\frac{h}{4}H_i\right) + \exp\left(\frac{-h}{4}H_i\right) \otimes X_i$$

$$\cdot\Delta(Y_i) = Y_i \otimes \exp\left(\frac{h}{4}H_i\right) + \exp\left(\frac{-h}{4}H_i\right) \otimes Y_i.$$

The antipode S is given by: $S(H_i) = -H_i$, $S(X_i) = -e^{h/2}X_i$, $S(Y_i) = -e^{-h/2}Y_i$.

This Hopf algebra is not cocommutative; the non-cocommutativity is measured by the so-called R-matrix, which "intertwines" Δ and the opposite comultiplication Δ' [1,2]. The images of R in tensor products of finite dimensional representations play an important role in the construction of representations of the braid group