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Fay's Trisecant Identity and Conformal Field Theory

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Abstract. We study the correlation functions of a system of free chiral fermions on a compact Riemann surface using techniques of algebraic geometry. Fay's trisecant identity arises as a consequence of the proof of the uniqueness of correlation functions.

1. Introduction

The remarkable identity discovered by Fay [1] (see also [2]), and known as the *trisecant identity*, has been found to play a fundamental role in the theory of Jacobian varieties [3]. In this paper we shall prove Fay's identity by studying a problem in *conformal field theory*. It has recently been observed by physicists that the trisecant identity arises naturally in the study of conformal field theories on a compact Riemann surface [4]. The trisecant identity was obtained in [4] as a result of computing the four point correlation function of a system of free chiral fermions on a compact Riemann surface by *bosonization* of the fermions, on the one hand, and by "*Wick's theorem*," on the other. An alternative approach, using Witten's idea of *multiplicative Ward identities* [5], was given in a recent paper by S. Sen and the present author [6] (see also [7]).

In the present work we make a detailed analysis of the geometry of the correlation functions of the free field theory of a conjugate pair ψ, $\tilde{\psi}$ of chiral fermions on a compact Riemann surface. The starting point of our analysis is the work of Friedan, Martinec, and Shenker [8] who have provided a local analysis of the correlation functions through operator product expansions. Their analysis provides us with the singularity structure (poles and zeros) of the correlation functions arising from the physics of the problem. *Our basic postulate is that the correlation functions have no poles other than those following from this analysis.*

We then study the global form of the correlation functions using techniques of algebraic geometry. We find that the line bundle of which a given correlation function is a meromorphic section has a very natural and simple structure. It is the tensor product of the line bundle which is defined by the singularity structure