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On Existence and Behaviour of Asymptotically Flat Solutions to the Stationary Einstein Equations

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Abstract. We show existence and uniqueness of asymptotically flat solutions to the stationary Einstein equations in $S=\mathbb{R}^3-B_r$, where B_r is a ball of radious $r>0$, when a small enough continuous complex function $\hat{u}$ on ∂S is given. Regularity and decay estimates imply that these solutions are analytic in the interior of S and also at infinity, when suitably conformally rescaled.

Introduction

After a considerable effort a rather clear picture describing the set of stationary, and asymptotically flat vacuum solutions to Einstein equations it is now available. Solutions are represented by a complex scalar field u, and a positive definite metric g_{ab} on a three dimensional manifold S [1–3]. From the four dimensional point of view this manifold is the quotient of space-time with the set of orbits of the killing vector field defining stationarity, the metric is conformally related to the one induced by the space-time metric on S, and u is a given functional of the norm, and the twist of the killing vector field. The equations they satisfy are,

$$(\Delta_g - 2R)u = 0, \tag{1}$$

$$\begin{aligned} G_{ab} &- 2(\nabla_{(a}u\nabla_{b)}u^* - (1+4|u|^2)^{-1/2}\nabla_a|u|^2\nabla_b|u|^2) \\ &- g_{ab}(\nabla_c u\nabla^c u^* - (1+4|u|^2)^{-1/2}\nabla_c|u|^2\nabla^c|u|^2) = 0, \end{aligned} \tag{2}$$

where G_{ab} is the three dimensional Einstein tensor corresponding to g_{ab}. We consider the following asymptotic boundary conditions $u \to 0$, $g_{ab} - e_{ab} \to 0$, as $r \to 0$, where e_{ab} is any flat metric on S, and r is the distance function with respect to it. Given a solution (u, g_{ab}) of the above equations it is possible to reconstruct a unique, stationary, asymptotically flat, maximally extended vacuum space-time.

From local elliptic theory [4, 5] we know that sufficiently smooth solutions (if they exist) are in fact analytic. Furthermore assuming a stronger asymptotic decay than the one above, one can show there exists a conformal factor such that the conformally rescaled fields are also sufficiently smooth and satisfy regular elliptic equations; thus they are also analytic, even at the point representing infinity