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A Note and Erratum Concerning "Min–Max Theory for the Yang-Mills–Higgs Equations"*

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Abstract. An error in an argument which was used to prove the existence of non-minimal solutions to the $SU(2)$ Yang-Mills–Higgs equations has been shown to the author. A revised proof is presented here to establish the existence of infinitely many non-minimal solutions to the afore-mentioned equations.

1. Introduction

In [T1], I described the topology of the configuration space of finite action pairs (A, Φ) of connection on the principal bundle $\mathbb{R}^3 \times SU(2)$ and section of the associated vector bundle $\mathbb{R}^3 \times$ Lie Alg $SU(2)$. The action functional which defines the topology is the Yang-Mills–Higgs action in the Prasad Sommerfield limit. The space of all finite action pairs, modulo the action of the gauge group $C^\infty(\mathbb{R}^3; SU(2))$, was denoted by B; and I proved that B was homotopy equivalent to the $\Omega^2(S^2)/S^1$, where $\Omega^2(S^2)$ is the space of smooth maps from S^2 to S^2 which take the north pole to itself and the group S^1 acts by rotating the image S^2 about the equator.

Associated to each configuration (A, Φ) is a Dirac operator coupled to the vector bundle $\mathbb{R}^2 \times \mathbb{C}^2$, and I showed that the assignment to each (A, Φ) of this Dirac operator defines a continuous map, δ, from B into the space of Fredholm operators. Proposition C3.1 of [T1] asserts that this map δ is homotopically non-trivial and pulls back non-zero cohomology of arbitrarily high degree from the space of Fredholm operators. Ralph Cohen has shown me an error in the proof of Proposition C3.1, and in fact, he has thrown considerable doubt onto its veracity.

Proposition C3.1 now sits unproved because Lemma C3.2 is erroneous. This lemma claims to construct an embedding of the configuration space C_n of unordered n-tuples of distinct points of $\mathbb{R}^3$ into the monopole number n component, B_n, of B. In fact, the construction in Definition C4.2 provides only an embedding of a fiber bundle over C_n, the fiber being $(x_n S^1)/S^1$, where S^1 acts on the n-torus diagonally. This fiber bundle has no continuous sections—the proof of Lemma C2.1 errs in assuming the existence of a section. This bundle is described for the interested reader at the end of this note.

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