Commun. Math. Phys. 121, 345–350 (1989)

Communications in Mathematical Physics

Cyclic Cocycles from Graded KMS Functionals

Daniel Kastler

Centre de Physique Théorique, Sect. 2, C.N.R.S., Luminy-Case 907, F-13288 Marseille Cedex 9, France

Abstract. Each "graded KMS functional" of a $Z/2$-graded C^*-algebra with respect to a "supersymmetric" one-parameter automorphism group gives rise to a cyclic cocycle.

In order to match algebras of primary mathematical interest for which there are no p-summable Fredholm modules, A. Connes introduced the wider notion of *θ-summable Fredholm module* [1], which also encompasses the Dirac operator on loop space rigorously constructed by A. Jaffe and collaborators [2] – and subsequently developed the corresponding generalizations of cyclic cohomology and of the Chern character [3]. For constructing the latter, Connes had to resort to a "formal square root" (Ref. [3], p. 20), so to speak enforcing supersymmetry, and thus leading to conjecture a deep relationship between cyclic cohomology, supersymmetry, and the modular theory of Von Neumann algebras [4]. On the other hand A. Jaffe, A. Lesniewski and K. Osterwalder were led by the investigation of supersymmetric field theoretical models [2] to propose (under a different name) an interesting alternative construction of the Chern character of a θ-summable Fredholm module [5] (cf. [9]).

The purpose of the present note is two-fold: first, using a $Z/2$-graded version of cyclic cohomology [6, 7], we enrich the (slightly adapted) Jaffe et al. (overall even) cocycle by a second component (odd both for the degree-of-form and the intrinsic grading)[1]. Second, we point out, as a first step towards the program [4], that the Jaffe et al. construction may be reinterpreted to pertain to "graded-KMS functionals" with respect to one-parameter automorphism groups "supersymmetric" in that they possess infinitesimal generators "with a square root." Under this aspect, [5] appears as describing the cocycle attached to the "superextension" of KMS-states of a type-I flavour. We defer to a later publication the discussion of more general cases.

[1] We in fact also treat the overall odd case (cf. 9 below)