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Breit-Wigner Formulas for the Scattering Phase and the Total Scattering Cross-Section in the Semi-Classical Limit

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Abstract. In this paper we prove results in resonance scattering for the Schrödinger operator $P_V = -h^2\Delta + V$, V being a smooth, short range potential on $\mathbf{R}^n$. More precisely, for energy λ near a trapping energy level λ_0 for the classical system defined by the Hamiltonian $p(x,\xi) = \xi^2 + V(x)$, we prove that the scattering phase and the scattering cross sections associated to (P_V, P_0) have the Breit-Wigner form ("Lorentzian line shape") in the limit $h \to 0$.

0. Introduction

We consider in this paper the semiclassical asymptotics of the scattering phase $s(\lambda, h)$ and the total scattering cross-section $\sigma(\omega, \lambda, h)$ associated to the Schrödinger operator $P = -h^2\Delta + V(x)$ on $\mathbb{R}^n$, for λ near a trapping energy level λ_0. By this, we mean that the classical flow associated to the Hamiltonian $p(x,\xi) = \xi^2 + V(x)$ has trapped trajectories in $p^{-1}(\lambda_0)$. Moreover, we will here assume that classical particles are trapped in $p^{-1}(\lambda_0)$ due to the presence of a potential well.

This configuration gives rise to the well known shape resonances for the quantum Hamiltonian, which have been extensively studied in the last few years: see for example the works of Helffer-Sjöstrand [He-Sj], Combes-Duclos-Klein-Seiler [C-D-K-S], and Hislop-Sigal [Hi-Si].

On the other hand, the problem of the short wave asymptotics for the scattering phase has been studied by many authors in optical and acoustical scattering problems. Let us mention for instance the works of Jensen-Kato [Je-Ka]; Majda-Ralston [Maj-Ra]; Petkov-Popov [Pe-Po] and Ivrii-Shubin [Iv-Sh].

For the Schrödinger operator, there are works of Colin de Verdière [CdV] and Guillopé [Gu] in the high energy limit, and of Robert-Tamura [Ro-Ta] in the semiclassical limit $h \to 0$.

However, all these results require a kind of non-trapping assumption in order to avoid problems caused by resonances (or poles of the S-matrix) close to the real axis.