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P-Adic Feynman and String Amplitudes

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Abstract. We derive an explicit representation for p-adic Feynman and Koba–Nielsen amplitudes and we briefly outline the connection between the scalar models of p-adic quantum field theory and Dyson's hierarchical models.

1. Introduction

As we have shown previously (see [1], submitted to "Theoretical and Mathematical Physics" in May 1987), the scalar models of the field theory over the p-adic field Q_p are the natural continuous analogs of Dyson's hierarchical models (see [2–5]). More precisely, the discretization of the field theory over Q_p on the hierarchical lattice of p-adic numbers with zero integer part is a model of Dyson's type. The traditional methods of quantum field theory such as Feynman diagrams, renormalization theory and Wilson's renormalization group have analogs in the p-adic case. The main results of [1] are briefly outlined in Sect. 2.

On the other hand, there has been recently some interest on the possibility of a p-adic formulation of string theory (see [5–12]).

All this explains our interest in the Feynman amplitudes over the p-adic field. The remarkable feature of p-adic models is the exact representation of Feynman and string scattering amplitudes as a sum of elementary functions. Namely, let us consider a general Feynman amplitude over Q_p in coordinate representation,

$$F(x_v; v \in V_{\text{ext}}) = \int \prod_{v,v' \in V_{\text{ext}} \cup V_{\text{int}}} \| x_v - x_{v'} \|_p^{a(v,v')} \prod_{v \in V_{\text{int}}} dx_v, \tag{1.1}$$

where the integral taken over $Q_p^{|V_{\text{int}}|}$, p is a fixed prime number, Q_p is a p-adic field, $\| \cdot \|_p$ is a p-adic norm (further on, the sign p will be omitted), V_{ext} (V_{int}) is a set of external (internal) vertices, $a(v, v') \in \mathbb{C}$ for each pair $v \in V_{\text{ext}} \cup V_{\text{int}}$, $v' \in V_{\text{ext}} \cup V_{\text{int}}$ (we identify a pair (v, v') with (v', v)), dx is a Haar measure on Q_p, normalized such

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