Commun. Math. Phys. 120, 665–688 (1989)

Communications in Mathematical Physics

Spin Glasses and Other Lattice Systems with Long Range Interactions

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Abstract. We study classical lattice systems, in particular real spin glasses with Ruderman-Kittel interactions and dipole gases, with interactions of very long (non-summable) range but variable sign. Using the Kac-Siegert representation of such systems and Brascamp-Lieb inequalities we are able to establish detailed properties of the high-temperature phase, such as decay of connected correlations, for these systems.

0. Introduction

In this paper we study the equilibrium statistical mechanics of classical spin systems with long-range exchange couplings of variable sign. A typical example of a system we propose to consider is a real spin glass with exchange couplings of Ruderman-Kittel (RKKY) type [1]. The Hamiltonian of such a system has the following structure:

$$H = -\sum_{i,j}\sum_{a,b} J_{ij}^{ab} n_i \sigma_i^a n_j \sigma_j^b - \sum_i h_i^3 n_i \sigma_i^3 . \tag{0.1}$$

Here i and j are sites of a lattice Γ (typically chosen to be $\mathbb{Z}^d$, $d=2,3,\ldots$); $\sigma_i=(\sigma_i^1,\ldots,\sigma_i^N)$, $N=1,2,3,\ldots$, is a classical spin variable at site i; n_i is a random variable taking the values 0 or 1 which indicates whether site i is occupied by a magnetic atom or ion ($n_i=1$) or by a non-magnetic one ($n_i=0$). The exchange couplings J_{ij}^{ab} are of long range and can be ferromagnetic or antiferromagnetic. We assume that they are the Fourier transforms of matrix-valued functions on the first Brillouin zone that are bounded in norm. As an example, we shall consider

$$J_{ij}^{ab} = \delta^{ab} \frac{1}{|i-j|+\lambda} \left(\frac{-k_F|i-j|\cos k_F|i-j| + \sin k_F|i-j|}{k_F|i-j|^3} \right). \tag{0.2}$$

Such models describe alloys of magnetic atoms or ions in a nonmagnetic host material, e.g. AuFe or CuMn.

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