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Perturbations of Gibbs Semigroups

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Abstract. We present the analytic perturbation theory for Gibbs semigroups in the case when perturbations of generators are relatively bounded. Analyticity with respect to perturbation and semigroup parameters in the Tr-norm topology is proved and the corresponding domains are described.

1. Introduction

Let $\mathfrak{C}_p(\mathfrak{H})$ be the Banach space of compact operators on a separable Hilbert space $\mathfrak{H}$ which have the finite $\|\cdot\|_p$-norm

$$\|A\|_p=\left(\sum_{n=1}^{\infty}\lambda_n^p\right)^{1/p}, \qquad 1\leqq p<\infty .$$

Here $\lambda_n=\mu_n(|A|)$, where $\mu_n(|A|)$ is a n^{th} (taking into account degeneracy) eigenvalue of the operator $|A|=\sqrt{AA^*}$. The Banach spaces $\{\mathfrak{C}_p(\mathfrak{H})\}_{p=1}^{\infty}$ are $*$-ideals in the space of all bounded operators $\mathscr{L}(\mathfrak{H})$ and $\mathfrak{C}_1(\mathfrak{H})$ *(trace-class)* $\subset\mathfrak{C}_2(\mathfrak{H})$ *(Hilbert-Schmidt operators)* $\subset\ldots\subset\mathfrak{C}_\infty(\mathfrak{H})$ *(compact operators)* $\subset\mathscr{L}(\mathfrak{H})$, see e.g. [1] or [2, VI.6].

In quantum statistical mechanics one faces strongly continuous one-parametric semigroups of self-adjoint operators $G:\mathbb{R}_+^1\cup\{0\}\to\mathscr{L}(\mathfrak{H})$ which have the property that $G:\mathbb{R}_+^1\to\mathfrak{C}_1(\mathfrak{H})$. They are naturally created by the *density matrix* $\exp(-\beta H)$ for a finite system with the Hamiltonian H and temperature $\beta^{-1}\in\mathbb{R}_+^1$ and got the name of the *Gibbs semigroups* [3–5]. But if we want to make an analytic continuation in the "interaction constant" then the operator H becomes non-self-adjoint. Its numerical range $\theta(H)=\{(H\psi,\psi):\ \psi\in D(H),\ \|\psi\|=1\}$ belongs to the sector $\mathscr{S}_\gamma(\Omega)=\{z\in\mathbb{C}:|\arg(z-\gamma)|\leqq\Omega<\pi/2\}$ and $G(t)=\exp(-tH)\in\mathfrak{C}_1(\mathfrak{H})$ for $t\in\mathbb{R}_+^1$, see e.g. [6, 7]. This was the reason for the following general definition.

Definition 1.1 [5]. A strongly continuous semigroup $G(t)$ in a separable Hilbert space $\mathfrak{H}$ is called a *Gibbs semigroup* if $G:\mathbb{R}_+^1\to\mathfrak{C}_1(\mathfrak{H})$.