

Pfaffian Bundles and the Ising Model

John Palmer*

Department of Mathematics, University of Arizona, Tucson, Arizona 85721, USA

Abstract. An infinite volume Pfaffian formalism is developed for the Ising model.

Introduction

In this paper we will establish a connection between the Pfaffian formalism for the Ising model and the transfer matrix formalism. In [H] Hurst makes the connection between the Pfaffian formalism and the transfer matrix formalism for the Ising model. The formalism he employs is not suited to a direct infinite volume analysis. Working with Grassmann integrals Sato, Miwa, and Jimbo [SMJ] have also worked out such a connection for a class of models they refer to as orthogonal models. The Grassmann integrals they employ only make sense in finite dimensions. Our main interest here is in formulating a direct connection in the thermodynamic limit where the relevant vector spaces are infinite dimensional.

The Pfaffian approach to the Ising model produces a formula for the partition function on a finite lattice as the Pfaffian (or sum of Pfaffians) of a finite dimensional skew symmetric matrix (see McCoy and Wu [MW]). In the infinite volume limit this skew symmetric matrix becomes a finite difference operator on $l^2(Z_{1/2}^2, R^4)$. The finite volume correlation functions are ratios of Pfaffians of operators with similar structure except that the numerator has inhomogeneities that depend on the n sites in the correlation function. It is difficult to rigorously control the infinite volume limit in the Pfaffian approach and we will not attempt to do so here. There is another approach to the Ising correlations where the thermodynamic limit has been rigorously treated [PT]. This is the original transfer matrix formalism of Onsager and Kaufmann [O, K]. In this paper we will start with the problem of understanding the Pfaffian for a family of skew symmetric operators on a Hilbert space, and we will then make the connection between this problem and the infinite volume transfer matrix formalism. We will have then established a Pfaffian

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