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Correlation Length Bounds for Disordered Ising Ferromagnets

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Abstract. The d-dimensional, nearest-neighbor disordered Ising ferromagnet:

$$\mathscr{H} = -\sum J_{ij}\sigma_i\sigma_j$$

is studied as a function of both temperature, T, and a disorder parameter, λ, which measures the size of fluctuations of couplings $J_{ij} \geqq 0$. A finite-size scaling correlation length, $\xi_f(T, \lambda)$, is defined in terms of the magnetic response of finite samples. This correlation length is shown to be equivalent, in the scaling sense, to the quenched average correlation length $\xi(T, \lambda)$, defined as the asymptotic decay rate of the quenched average two-point function. Furthermore, the magnetic response criterion which defines ξ_f is shown to have a scale-invariant property at the critical point. The above results enable us to prove that the quenched correlation length satisfies:

$$C|\log \xi(T)|\, \xi(T) \geqq |T - T_c|^{-2/d},$$

which implies the bound $\nu \geqq 2/d$ for the quenched correlation length exponent.

1. Introduction

The behavior of the correlation length exponent for disordered systems has been a subject of interest for some time. In 1974, Harris [2] suggested a simple criterion to determine whether or not the critical behavior of a given system is affected by a small amount of disorder. He argued that if the correlation length exponent, ν_p, of a d-dimensional pure (uniform) system satisfies the bound $\nu_p > 2/d$, then disorder is irrelevant in the renormalization group sense and thus should not change the critical behavior. A natural generalization of Harris' result (which does not, however, follow directly from his line of argument) is that in *all* disordered systems with continuous transitions, the correlation length exponent ought to satisfy the bound

$$\nu \geqq \frac{2}{d}. \tag{1.1}$$

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