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Capture and Confinement of Solitons in Nonlinear Integrable Systems

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Abstract. The system describing the interaction of a long wave with a short wave packet on the x, y plane is considered. The solutions are found to describe a soliton that comes from infinity and then is captured into a conditionally periodical oscillatory regime. The solutions are also found that describe a soliton coming from infinity and then decaying into two solitons: one goes to infinity and the other is captured into a conditionally periodical oscillatory regime. The obtained results are relevant to some problems of hydrodynamics, plasma physics, solid state physics, etc.

Introduction

The present paper is devoted to the problems of capture and confinement of solitons in nonlinear integrable systems. More exactly, the case in point is the following phenomenon. Nonlinear integrable systems considered below have been found to have solutions describing a soliton that comes from infinity, then is captured into a conditionally periodical oscillatory regime and remains in this state in all subsequent times. These systems also have solutions describing a soliton that executed a conditionally periodical oscillatory motion, then came off and went to infinity. Further, we have found solutions describing the soliton that comes from infinity, then decays into two solitons: one being captured into the conditionally periodical oscillatory regime and the other going to infinity. Making a time inversion in these solutions, we get new solutions describing the soliton coming from infinity which collides with the other soliton that has been in the regime of conditionally periodical oscillatory motion; as a result of the collision these two solitons fuse into one soliton that goes to infinity.

We proceed from the following system of equations:

$$\begin{aligned} &3\frac{\partial^2 v}{\partial y'^2} - \frac{\partial}{\partial x'}\left[\frac{\partial v}{\partial t'} + \frac{\partial}{\partial x'}\left(3v^2 + \frac{\partial^2 v}{\partial x'^2} + 8\kappa|\psi|^2\right)\right] = 0, \\ &ic_0\frac{\partial \psi}{\partial t'} + ic_1\frac{\partial \psi}{\partial x'} + i\frac{\partial \psi}{\partial y'} = v\psi + \frac{\partial^2 \psi}{\partial x'^2} + c_2\frac{\partial^2 \psi}{\partial y'^2} + c_3|\psi|^2\psi \end{aligned} \tag{1}$$