Commun. Math. Phys. 120, 353–378 (1989)

Communications in Mathematical Physics

String Structures on Loop Bundles

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Abstract. Differential geometry and topology of principal loop bundles (bundles of loop groups over loop spaces) are investigated. String structures, defined as bundle extensions corresponding to the central extension of the structure group, do not always exist. Various methods for deriving the obstruction to the existence of string structures are discussed.

1. Introduction

Recently, Killingback [1] introduced a notion of string structure and suggested that it should play a similar role in defining the Dirac-Ramond operator on the loop space, LM, of a compact riemannian manifold, M, as the spin structure does in the case of the usual Dirac operator on M.

For a principal fibre bundle, LP, over LM, with a structure loop group, LG, the string structure is defined as a bundle extension of LP to $\hat{L}P$, whose structure group is the central extension of LG. In cases which are interesting in applications to string theory the bundles on the loop space are obtained from some bundles over the space itself, for example, from the spin bundle of M or some G bundle which corresponds to the background gauge fields on M. It has been also observed in [1] that in such cases the topological conditions, which one derives by requiring that a string structure exists, are similar to those that guarantee the anomaly cancellation in the two-dimensional chiral supersymmetric σ-model on M, coupled to gauge and gravitational backgrounds. This turns out to be quite natural once one realizes the role of the supersymmetric σ-models in defining the Dirac-type operators [2–6]. For example, it is known that the usual Dirac operator corresponds to the supersymme-

* Supported in part by the U.S. Department of Energy under Grant #DE-FG02-84ER40158 with Harvard University.
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** Supported in part by the NSF Grant #87-08447 and by the U.S. Department of Energy under Contract #DE-AC02-76ER-03069.
On leave from the University of Wroclaw, Wroclaw, Poland