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Euclidean Formulation of Quantum Field Theory Without Positivity

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Abstract. General properties of local quantum field theories (QFT) without positivity are discussed in connection with their euclidean formulation. Modified euclidean axioms for local QFT's without positivity are presented, which allow us to recover by analytic continuation Wightman functions satisfying the modified Wightman axioms for indefinite metric QFT's.

1. Introduction

With the advent of gauge theories it became clear that it was natural (if not necessary) to consider QFT in which not all the Wightman axioms are satisfied. In particular, it appeared that the introduction of "charged" fields was in conflict with either locality or positivity [1–4]. On the other hand, the success of (perturbative) renormalization theory (also for gauge theories) [5, 6] and the usefulness of keeping a relation with the wisdom gathered from conventional perturbation theory made clear that it could be better, at least at a technical level, to keep locality rather than positivity [3, 4]. Actually, it turned out that even the solution of a long-standing problem like the infrared problem [7] and the construction of charged states in QED was made possible by exploiting the local structure, in a spirit close to the standard Wightman formulation [8]. Even the recent deep results about the geometrical understanding of anomalies in QFT have been made possible by a formulation which kept locality as a basic structure [9, 10]. Also recent attempts of a quantum field theory formulation of string theories with emphasis on the "covariant gauges" suggest that it may be of some interest to investigate the general properties of indefinite metric quantum field theories. Finally, it need not be emphasized here that most of the wisdom gained (on a heuristic level) about "non-perturbative" treatment of covariant gauge field theories and/or covariant string theories heavily rely on the use of the so-called "functional integral techniques," namely of the euclidean formulation of the theory. To our knowledge, a careful discussion of the euclidean formulation of quantum field

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