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Global Existence for the Relativistic Vlasov–Maxwell System with Nearly Neutral Initial Data

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Abstract. Global classical solutions to the initial value problem for the relativistic Vlasov–Maxwell equations are obtained in three space dimensions. The initial distribution of the various species may be large, provided that the total positive charge nearly cancels the total negative charge.

Introduction

Consider a plasma which consists of a large number of charged particles. We assume there are N distinct species of charge and that particles of the α^{th} species have charge e_α and rest mass m_α. Their distribution in phase space at time t is $f_\alpha(t, x, v)$, where $x \in \mathbb{R}^3$ represents position and $v \in \mathbb{R}^3$ momentum. Thus the charge and current densities are

$$\rho(t, x) = 4\pi \int \left(\sum_\alpha e_\alpha f_\alpha \right) dv$$

and

$$j(t, x) = 4\pi \int \left(\sum_\alpha e_\alpha f_\alpha \hat{v}_\alpha \right) dv,$$

where

$$\hat{v}_\alpha = (m_\alpha^2 + c^{-2}|v|^2)^{-1/2} v$$

gives the velocity of a particle with momentum v and rest mass m_α (c is the speed of light). The state of the plasma at time t is given by $f_\alpha(t, \cdot, \cdot)$ for $\alpha = 1, \ldots, N$ and $E(t, \cdot)$, $B(t, \cdot)$, where E and B are the electric and magnetic fields. If electromagnetic effects dominate collisional effects, we may model the time evolution of the plasma with the system:

$$\text{(RVM)} \begin{cases} \partial_t f_\alpha + \hat{v}_\alpha \cdot \nabla_x f_\alpha + e_\alpha (E + c^{-1} \hat{v}_\alpha \wedge B) \cdot \nabla_v f_\alpha = 0 \\ \partial_t E = c \nabla \wedge B - j, \quad \nabla \cdot E = \rho \\ \partial_t B = -c \nabla \wedge E, \quad \nabla \cdot B = 0 \end{cases}$$

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