Commun. Math. Phys. 119, 95–108 (1988)

Communications in Mathematical Physics

A Nekhoroshev-Type Theorem for Hamiltonian Systems with Infinitely Many Degrees of Freedom

Giancarlo Benettin,[1] Jürg Fröhlich[2] and Antonio Giorgilli[3]

[1] Dipartimento di Fisica dell'Università di Padova, Via Marzolo 8, I-35131 Padova, Italy
[2] Theoretical Physics, ETH-Hönggerberg, CH-8093 Zürich, Switzerland
[3] Dipartimento di Fisica dell'Università di Milano, Via Celoria 16, I-20133 Milano, Italy

Abstract. We study the propagation of lattice vibrations in models of disordered, classical anharmonic crystals. Using classical perturbation theory with an optimally chosen remainder term (i.e. a Nekhoroshev-type scheme), we are able to show that vibrations corresponding to localized initial conditions do essentially not propagate through the crystal up to times larger than any inverse power of the strength of the anharmonic couplings.

1. Introduction: General Ideas and Main Results

In this paper we study the trapping, or localization of lattice vibrations in models of classical, disordered anharmonic crystals. Localization is a phenomenon observed in the study of wave propagation in disordered media. When the disorder is large enough waves can get trapped or propagate anomalously slowly. This implies that there is no transport of energy in such a system.

Anderson first studied localization in the motion of quantum mechanical electrons through a random array of scatterers [1]. This is relevant for an analysis of electrical conductivity in disordered metals. Anderson's arguments have been made rigorous in [2] (for one-dimensional systems) and [3] (higher dimensional systems). Electron waves are linear waves whose propagation is governed by the Schrödinger equation. In contrast, the transport of heat or sound through bulk matter, or of electromagnetic waves through a non-linear medium, poses a problem in non-linear wave propagation. One would like to understand, for example, the transport properties of a disordered, anharmonic crystal. A conventional idea is that if the disorder is large enough there is no transport of energy in such a system, or transport is anomalously slow. A proof of this conjecture would require mathematical control of wave propagation over arbitrarily long time scales. For a special class of models of classical anharmonic crystals and some very special initial conditions, such control has been achieved in [4]. But the results in [4] do not permit one to establish bulk transport properties of those model systems.

Real or computer experiments are performed in a finite, though possibly long time interval. One might argue, therefore, that it is of interest to study wave