

Construction of Analytic KAM Surfaces and Effective Stability Bounds

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To our friend and colleague Paola Calderoni

Abstract. A class of analytic (possibly) time-dependent Hamiltonian systems with d degrees of freedom and the “corresponding” class of area-preserving, twist diffeomorphisms of the plane are considered. Implementing a recent scheme due to Moser, Salamon and Zehnder, we provide a method that allows us to construct “explicitly” KAM surfaces and, hence, to give lower bounds on their breakdown thresholds. We, then, apply this method to the Hamiltonian $H \equiv y^2/2 + \varepsilon(\cos x + \cos(x-t))$ and to the map $(y, x) \rightarrow (y + \varepsilon \sin x, x + y + \varepsilon \sin x)$ obtaining, with the aid of computer-assisted estimations, explicit approximations (within an error of $\sim 10^{-5}$) of the golden-mean KAM surfaces for complex values of ε with $|\varepsilon|$ less or equal than, respectively, 0.015 and 0.65. (The experimental numerical values at which such surfaces are expected to disappear are about, respectively, 0.027 and 0.97.) A possible connection between break-down thresholds and singularities in the complex ε -plane is pointed out.

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* Supported by Consiglio Nazionale delle Ricerche, Italy

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