

## Billiards with Pesin Region of Measure One

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**Abstract.** We give a description of a large class of plane billiards with Pesin region of measure one. Open conditions including properly those founded by Wojtkowski [W1] for  $C^4$  focusing boundaries are obtained. Lyapunov’s forms, introduced by Lewowicz, are used.

### 0. Introduction

Boltzmann’s hypothesis about the ergodicity of the model (Boltzmann-Gibbs) of the gas of hard spheres is in the basis of the study of the statistical properties of dynamical systems. Sinai’s proof [S1] of the simplest case of this hypothesis concerns the plane with two balls and a square box represented by the torus  $\{(x, y) \bmod 1\}$ . This situation is immediately extended for billiards with boundaries of negative curvature (dispersing billiards).

Birkhoff, Krylov, and Arnold gave intuitive interpretations of the relations between the dynamical systems of the billiard type and the geodesic flows of surfaces. The study of ergodic properties of these flows in surfaces of negative curvature showed the importance of the transversal stable and unstable foliations (separately, Hopf and Hedlund, 1939). This methods were profitably used since the sixties in the study of the so-called Anosov systems and other topics [see for example (A1)].

The above mentioned relations justify the use of the same techniques in the study of ergodic properties of billiards with dispersing boundaries. This is the central idea of Sinai’s work.

In the billiards with focusing boundaries the situation is more complicated and the pioneer works of Birkhoff suggested the non-ergodicity of plane billiards with boundaries that consist of a smooth curve of positive curvature. Bunimovich [B2, B3], indicated that billiards whose focusing pieces of the boundary have constant curvature and that do not contain dispersing components, are Bernouilli (the stadium, for example).