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The Large Deviation Principle and Some Models of an Interacting Boson Gas

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Abstract. This is a study of the equilibrium thermodynamics of the Huang-Yang-Luttinger model of a boson gas with a hard-sphere repulsion using large deviation methods; we contrast its properties with those of the mean field model. We prove the existence of the grand canonical pressure in the thermodynamic limit and derive two alternative expressions for the pressure as a function of the chemical potential. We prove the existence of condensate for values of the chemical potential above a critical value and verify a prediction of Thouless that there is a jump in the density of condensate at the critical value. We show also that, at fixed mean density, the density of condensate is an increasing function of the strength of the repulsive interaction. In an appendix, we give proofs of the large deviation results used in the body of the paper.

1. Introduction

Since London's proposal [1] that the super-fluid phase-transition in He^4 is an example of Bose-Einstein condensation, it has been of interest to know how, in theory, interparticle forces affect the condensation of bosons. London himself conjectured [2] that, as a manifestation of quantum mechanical complementarity, the momentum-space condensation of bosons is enhanced by spatial repulsion between the particles; we know of no proof of this general proposition.

Huang et al. [3] introduced a model of a boson gas with a hard-sphere repulsion which displays enhanced condensation. The model may be described thus: Let $\Lambda_1, \Lambda_2 \ldots$ be a sequence of regions in $\mathbb{R}^d$ and denote the volume of Λ_l by V_l; we assume that $V_l \to \infty$ as $l \to \infty$. We associate with the region Λ_l the sequence $\varepsilon_l(1) \leqq \varepsilon_l(2) \leqq \ldots$ of ordered real numbers, where $\varepsilon_l(j)$ is the j^{th} eigenvalue of the single particle hamiltonian of the non-interacting system in the region Λ_l; the free-gas hamiltonian H_l^0 is given by

$$H_l^0 = \sum_{j \geqq 1} \varepsilon_l(j) n_l(j), \tag{1.1}$$