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Quantum K-Theory

I. The Chern Character*

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Abstract. We construct a cocycle on an infinite dimensional generalization of a p-summable Fredholm module. Our framework is related to Connes' cyclic cohomology and is motivated by our work on index theory on infinite dimensional manifolds. The p-summability condition is characteristic of dimension $O(p)$. We replace this assumption by the requirement that there exists an underlying heat kernel which is trace class. Then we use the heat kernel to regularize states in dimension-independent fashion. Our cocycle may be interpreted as an infinite dimensional Chern character.

I. Introduction

This paper continues our work on index theory on infinite dimensional manifolds [8–12] and connects it with Connes' theory of cyclic cohomology [2–4]. Our aim is to formulate non-commutative differential geometry in an infinite dimensional setting. We define the notion of a quantum algebra and formulate a cohomology theory for such algebras. Our framework is similar to Connes' theory of Θ-summable Fredholm modules [4].

Non-trivial, infinite-dimensional examples of these structures arise from existence theorems for two-dimensional, supersymmetric, i.e. $\mathbb{Z}_2$-graded quantum fields. In this case, the underlying, infinite-dimensional manifold is the loop space of a finite-dimensional manifold M, namely the space ΛM of smooth maps from the circle to M,

$$\Lambda M = \{\varphi : S^1 \to M\}. \tag{I.1}$$

The field theory examples constructed so far [8–12] arise from the choices $M = \mathbb{C}$ and $M = \mathbb{R}$. They yield a Hilbert space

$$\mathscr{H} = L_2(\Lambda M) \oplus L_2(\Lambda M),$$

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