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Another Construction of the Central Extension of the Loop Group

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Abstract. By considering the geometry of the central extension of the loop group as a principal bundle it is shown that it must be the quotient of a larger group. This group is a central extension of the group of paths in the loop group and its cocycle is constructed as the holonomy around a certain path. Conversely it is shown that this definition of a cocycle gives a method of constructing the central extension. The Wess-Zumino term plays an important role in these constructions.

1. Introduction

Mickelsson (1987) [and see also Frenkel (1986)] gives a remarkably simple construction of the central extension $U(1)\to\hat{\Omega}G\to\Omega G$ of the loop group ΩG. It is well known that that the fibering $\hat{\Omega}G\to\Omega G$ is topologically non-trivial so that $\hat{\Omega}G$ cannot be constructed as $U(1)\oplus\Omega G$ with a new group multiplication $(g,\lambda)(h,\mu)=(gh,c(g,h)\lambda\mu)$ for some continuous cocycle $c(g,h)$. What Mickelsson does, however, is to consider a larger group DG which has ΩG as a quotient. He then shows that there is a cocycle which defines a central extension of DG and a normal subgroup of $DG\oplus U(1)$ such that the quotient group is $\hat{\Omega}G$.

By using the path fibration of the loop group another such construction is possible which gives rise to a different cocycle as the holonomy, or Wess-Zumino term, for a particular closed path in the loop group.

In this method the larger group DG arises as the group of paths and its central extension as a group of horizontal paths. It is easy to see then that it has the loop group central extension as a quotient and to identify the kernel as the loops in the loop group.

Conversely it is also readily shown that with this form of the cocycle DG has a central extension with a naturally defined normal subgroup and that the quotient gives a method of constructing the central extension of the loop group.

2. Invariant Connections

For the convenience of notation let $\mathscr{G}$ denote the loop group. In fact it could be any group with a central extension

$$U(1)\to\hat{\mathscr{G}}\xrightarrow{\pi}\mathscr{G}.$$