Commun. Math. Phys. 116, 1–22 (1988)

Communications in Mathematical Physics

Renormalization Group Approach to Lattice Gauge Field Theories
II. Cluster Expansions*

Tadeusz Balaban
Department of Mathematics, Boston University, Boston, MA 02215, USA

Abstract. The fluctuation field integral, constructed in Part I, is represented by the exponentiated cluster expansion. It is proved that the terms of the expansion satisfy the inductive assumptions. This completes the construction of the sequence of effective actions in the small field approximation.

Introduction

In the first paper of this series we have considered the fluctuation field integral defined by the k^{th} renormalization transformation. We have shown there that the fluctuation field effective action is a small perturbation of the basic quadratic form, in the small field approximation. This is the main part of the analysis of this integral, and it includes the analysis of renormalization. Now it remains to construct a localized representation for the new term in the effective action defined by the fluctuation field integral. This is done by an application of the exponentiated cluster expansion. This expansion is constructed in two steps.

At first we localize the fluctuation field effective action by a procedure similar to a cluster expansion, using the generalized random walk expansions for propagators and minimizers occurring in the action. This procedure is described in Sect. 1. It yields the integral in the form to which we can apply in a straightforward way the exponentiated cluster expansion. This expansion is described in Sect. 2. It yields the desired localized expansion of the new effective action. We prove also that terms of this expansion satisfy the inductive assumptions formulated in the first paper. Thus we complete the proof of Theorem 3 of that paper. The two expansions constructed here are quite general, as will become clear from their descriptions. They can be, and will be, applied in many other situations, such as for the expressions constructed with the help of more general propagators described in [13], or for integrals conditioned to subdomains of the lattice.

* Work supported in part by the Air Force under Grant AFOSR-86-0229 and by the National Science Foundation under Grant DMS-86-02207