

# Analytic Torsion and Holomorphic Determinant Bundles I. Bott–Chern Forms and Analytic Torsion

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**Abstract.** We attach secondary invariants to any acyclic complex of holomorphic Hermitian vector bundles on a complex manifold. These were first introduced by Bott and Chern [Bot C]. Our new definition uses Quillen’s superconnections. We also give an axiomatic characterization of these classes. These results will be used in [BGS2] and [BGS3] to study the determinant of the cohomology of a holomorphic vector bundle.

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## Introduction

This is the first of a series of three papers, which are devoted to the study of the determinant line bundle of the direct image of a holomorphic vector bundle. Parts II and III of this work will be referred to as [BGS2] and [BGS3]. We first summarize the results which are obtained in these papers.

Let  $\pi: M \rightarrow B$  be a proper holomorphic map of complex analytic manifolds and let  $\xi$  be a complex holomorphic vector bundle on  $M$ .

According to Grothendieck and Knudsen–Mumford [KM], the (derived) direct